Eur. Phys. J. B **7**, 175–178 (1999)

THE EUROPEAN PHYSICAL JOURNAL B

EDP Sciences

Rapid Note

Specific heat anomaly and adiabatic hysteresis in disordered electron systems in a magnetic field

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Received: 17 July 1998 / Revised: 3 September 1998 / Accepted: 5 October 1998

Abstract. We consider the thermodynamic behavior of a disordered interacting electron system in two dimensions. We show that the corrections to the thermodynamic potential in the weakly localized regime give rise to a non monotonic behavior of the specific heat both in temperature and magnetic field. From this effect we predict the appearance of adiabatic hysteresis in the magnetoconductance. Our results can be interpreted as precursor effect of formation of local moments in disordered electron systems. We also comment on the relevance of our analysis in three dimensional systems.

PACS. 71.30.+h Metal-insulator transitions and other electronic transitions – 72.15.Rn Quantum localization – 73.20.Fz Weak localization effects (e.g., quantized states) – 73.40.Qv Metal-insulator semiconductor structures (including semiconductor-to-insulator)

The recent discovery of a $B = 0$ metal-insulator transition (MIT) in high mobility silicon MOSFETs [1] has revived the interest in disordered electron systems. Whereas conventional scaling theory predicts no metallic state in two dimensions [2] the experiments show a metallic temperature dependent resistivity with a characteristic exponential behavior.

Although there have been attempts to interpret these experimental findings within a phenomenological single parameter scaling theory [3], there is growing experimental evidence as witnessed by magnetic field measurements [4] that electron-electron interaction in the spin channel is relevant and needs explicit consideration. Based on the existing theory of disorder and interaction [5], it has been pointed out recently [6] that the very existence of a metallic state at low temperatures is indeed possible because of enhanced spin fluctuations, though a more thorough understanding of the behavior of various physical quantities is doubtless required. In particular it has been suggested that magnetoconductance in parallel field and tunneling measurements should provide good diagnostic tests for this theory.

A further distinctive prediction of the theory of disorder and interaction concerns the anomalous low temperature behavior of the thermodynamical quantities [7,8]. This appears to be consistent, at least qualitatively, with a number of experimental results, among which NMR, spin susceptibility, and specific heat measurements in 3d Si:P metallic samples [9]. At phenomenological level, a two fluid picture consisting of a free electron gas coupled to a set of localized magnetic moments seems to capture the main features of the experimental results.

From the theoretical point of view the possibility of formation of local moments by increasing disorder has been suggested since the early developments of the theory of the combined effects of disorder and interaction [7]. In 3d the renormalization group (RG) flows to a Fermi liquid if disorder is sufficiently weak. However, by suitably tuning the couplings of the RG one finds that indeed the combined effect of disorder and interaction leads to a magnetic instability signaling the formation of magnetized regions [7]. An alternative point of view associates the local moments in 3d Si:P to the presence of rare fluctuations [10]. The relation and interplay between the two proposed mechanisms of local moment formation is still an open issue.

Within the scaling theory the magnetic instability occurs, in 2d, for arbitrarily weak disorder and one argues that local moments form on a finite length scale at which the magnetic susceptibility diverges and the RG stops [7]. 2d systems then, apart from being presently of great interest, are good candidates for testing the RG approach to the interaction and disorder effects in the thermodynamics.

In this paper we revisit the thermodynamic behavior of a two-dimensional disordered interacting electron system. In particular we show that by taking into account the correction to the thermodynamic potential due to the combined effect of disorder and interaction in the particle-hole

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spin channel (triplet channel), there arises a non monotonic behavior of the specific heat both in temperature and magnetic field, strongly resembling the Schottky anomaly of free local moments. We interpret this as a dynamical precursor effect of formation of local moments. The above non monotonic behavior may well be difficult to observe by a direct measurement of the specific heat of the 2d system. We then propose to perform magnetoconductance measurements in a time varying magnetic field and look for hysteresis (more details below) [11]. We shall suggest to carry out this experiment well inside the metallic phase, for which we shall give predictions for the main energy scales.

The starting point is the expression of the thermodynamic potential F in the presence of a magnetic field. We consider the corrections coming from the singlet and triplet particle-hole channels in the weakly localized regime, *i.e.* in lowest order in the dimensionless resistance. We neglect corrections from the particle-particle (Cooper) channel and orbital effects [12]. The magnetic field is coupled *via* the Zeeman splitting of the spin states. These corrections were calculated in reference [13] for weak interactions and extended to strong scattering amplitudes in reference [14]. Here, for completeness, we further extend these calculations to include the energy renormalization Z which plays the role of m^*/m in the context of the Fermi liquid theory of disordered systems [8,15]. The singlet and triplet particle-hole contribution to F is $\delta F = \sum_{J,M} F_M^J$ where J and M are the total and the z component of the spin of the particle-hole pair (z being the direction of the applied magnetic field). J takes the values $J = 0, 1$ for the singlet and triplet contributions, respectively.

In 2d the quantities F_M^J are

$$F_0^0 + F_0^1 = gN_{QP}(-1+\gamma_2) \times \int_0^{\tau^{-1}} d\omega \left(b\left(\frac{\omega}{T}\right) + \frac{1}{2}\right) \omega \log(\omega\tau) \quad (1)$$

and

$$\sum_{M=\pm1} F_M^1 = gN_{QP} \int_0^{\tau^{-1}} d\omega \left(b\left(\frac{\omega}{T}\right) + \frac{1}{2}\right) \left[\omega \log\left|\frac{\omega^2-\Omega_s^2}{\omega^2-\tilde{\Omega}_s^2}\right| + \gamma_2 \omega \log|(\omega^2-\Omega_s^2)\tau^2| + \tilde{\Omega}_s \log\left|\frac{(\omega+\Omega_s)(\omega-\tilde{\Omega}_s)}{(\omega-\Omega_s)(\omega+\tilde{\Omega}_s)}\right|\right]. \quad (2)$$

Here $g = 1/(4\pi^2 N_0 D) = e^2/(\pi h)R_\square$ is the dimensionless resistance in two dimensions, D is the diffusion coefficient, $N_{QP} = ZN_0$ is the quasiparticle density of states per spin (N_0 being the bare density of states), τ is the elastic scattering time and $b(\omega) = [\exp(\omega) - 1]^{-1}$ is the Bose function. Equations (1, 2) are valid in the case of long range Coulomb forces with γ_2 being the interaction coupling constant in the triplet particle-hole channel [16]. Finally, $\Omega_s = g_L \mu_B H$ and $\tilde{\Omega}_s = (1+\gamma_2)\Omega_s$ are the bare and interaction dressed Zeeman spin splitting frequencies, which enter the diffusive particle-hole propagators [14,17].

One can rewrite δF by separating the temperature and the magnetic field dependent parts as follows

$$\delta F = F_0(T) + F_1(\Omega_s) + gN_{QP}T^2 f_2\left(\frac{\Omega_s}{T}\right) \quad (3)$$

where

$$F_0(T) = -gN_{QP}(1-3\gamma_2)T^2\left(\frac{\pi^2}{6}\log(\tau T) + a\right), \quad (4)$$

$$F_1(\Omega_s) = \frac{1}{2} gN_{QP}\gamma_2(1+\gamma_2)\Omega_s^2 \log \Omega_s \tau, \quad (5)$$

$$f_2\left(\frac{\Omega_s}{T}\right) = \left((1+\gamma_2) f\left(\frac{\Omega_s}{T}\right) - f\left(\frac{\tilde{\Omega}_s}{T}\right)\right) - 2\gamma_2 a \quad (6)$$

with $a = \int_0^\infty dy\, y b(y) \log y \approx -0.24$ and

$$f(x) = \int_0^\infty dy\, b(y)\left[(y-x)\log|y-x| + (y+x)\log|y+x|\right]. \quad (7)$$

At small x, $f_2(x) \approx -(1/2)\gamma_2(1+\gamma_2)x^2 \log x$.

At zero magnetic field, equation (4) gives the leading logarithmic correction to the specific heat $c_V = -T\partial^2 F/\partial T^2$, which then is logarithmically enhanced at low temperatures for $\gamma_2 > 1/3$. This correction signals the breakdown of perturbation theory for the quasiparticle density of states and is related to the scaling equation for the energy renormalization Z [8,18]. The analysis of the RG equations in two dimensions shows that upon scaling both Z and γ_2 grow so rapidly that the renormalization procedure must stop at a finite length scale, $L_c = l \exp[c/g_0(\gamma_{2,0}+1)]$, where they both diverge. l is the mean free path, c a number of order one, and g_0, $\gamma_{2,0}$ are the bare values of the running couplings g and γ_2. Because Z and γ_2 are related to the specific heat and to the spin susceptibility *via* the relations $c_V/c_V^0 = Z$ and $\chi/\chi_0 = Z(1+\gamma_2)$, this divergence signals a magnetic instability and the above length scale L_c has been interpreted as the typical size over which local moments form in the disordered system [7]. The very existence of local moments is expected to have specific consequences in the behavior of thermodynamic quantities, in particular in the presence of a magnetic field. In the case of free local moments, the specific heat shows the so-called Schottky anomaly, which manifests as a non monotonic behavior both in temperature and in magnetic field. On the experimental side, the behavior in the specific heat and magnetic susceptibility has provided some support to the idea of formation of local moments in Si:P materials [9].

In the theory of disordered systems, a finite magnetic field enters as a mass term in the diffusive particle-hole triplet propagators with $M = \pm 1$, thus effectively cutting off the logarithmic singularities. As a consequence, the weak localization correction to the specific heat, δc_V, is a decreasing function of the magnetic field H, at least when $H \gg T$. However, explicit evaluation of the leading

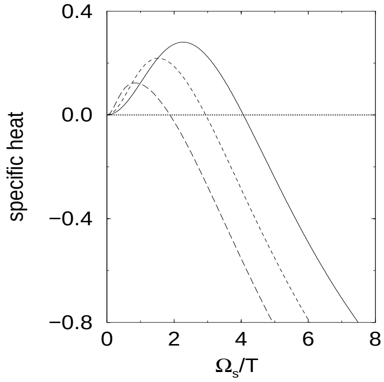

Fig. 1. Magnetic field dependent specific heat, $\Delta c_V(T,\Omega_s)/[c_V g\gamma_2]$ *vs.* Ω_s/T. The full, dashed and long-dashed lines represent $\gamma_2 = 1, 5, 10$.

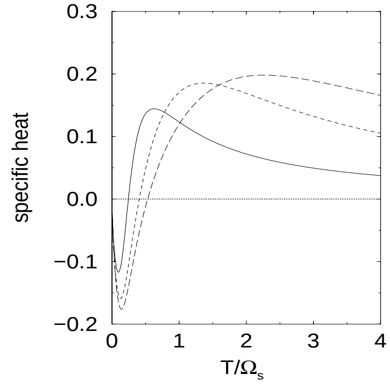

Fig. 2. Temperature dependence of the specific heat in presence of a magnetic field $\Delta c_V(T,\Omega_s)/[c_V g\gamma_2](T/\Omega_s)$ *vs.* T/Ω_s. The full, dashed, long-dashed lines represent $\gamma_2 = 1, 5, 10$.

$(H/T)^2$ term at small H/T shows that δc_V increases with the field, resembling the Schottky anomaly for free spins. It is useful to define the specific heat relative to the case with zero magnetic field

$$\Delta c_V(T,\Omega_s) = c_V(T,\Omega_s) - c_V(T,0) = -gN_{QP}T \times \left(2f_2\left(\frac{\Omega_s}{T}\right) - 2\frac{\Omega_s}{T}f_2'\left(\frac{\Omega_s}{T}\right) + \left(\frac{\Omega_s}{T}\right)^2 f_2''\left(\frac{\Omega_s}{T}\right)\right) \tag{8}$$

where f_2 and its derivatives can be evaluated numerically from equations (6, 7). The result for Δc_V normalized to $c_V g\gamma_2$ with $c_V = (2/3)\pi^2 N_{QP}T$ is shown in Figure 1. The quantity $\Delta c_V(T,\Omega_s)$ has a non monotonic behavior as function of the magnetic field, resembling the Schottky anomaly of free local moments. The values $\Omega_{s,max}$ and $\Omega_{s,0}$ at which Δc_V is maximum and zero, respectively, depend on γ_2 and move to smaller values upon increasing γ_2. A non monotonic behavior is also present in the temperature dependence. This is shown in Figure 2 where we report $(\Delta c_V(T,\Omega_s)/(c_V g\gamma_2))(T/\Omega_s)$ *versus* T/Ω_s.

Introduction of a magnetic field results in $\Delta c_V > 0$ ($\Delta c_V < 0$) for $T > \Omega_{s,0}$ ($T < \Omega_{s,0}$). This should be contrasted with the case of free local moments where $\Delta c_V > 0$ always. A magnetic field dependent specific heat is a direct consequence of the magnetic field dependence of the entropy. For free local moments this dependence leads to a non monotonic $(\partial S/\partial H)_T$ decreasing linearly at small H. We find that the excess entropy $\Delta S = S(T,H) - S(T,0)$ which follows from equation (2) has a field dependence which mimics that of local moments. In Figure 3 we report $\partial S(T,H)/\partial H$ *vs.* H/T normalized to $c_V g\gamma_2/T$. We also report $\partial S_{loc}(T,H)/\partial H$ for local moments. We rescale the density of local moments to fix the slope at small H to the slope of $\partial S(T,H)/\partial H$ for $\gamma_2 = 3$. Like in the free local moment system, $\partial S(T,H)/\partial H$ shows a minimum, whose location $\Omega_{s,min}$ depends on the strength of the interaction. Notice that for $\gamma_2 \approx 3$ one obtains $\Omega_{s,min}/T \approx 1.5$, close to the value for free local moments.

If local moments are thermally well coupled to the conduction electrons and weakly coupled to a heat bath,

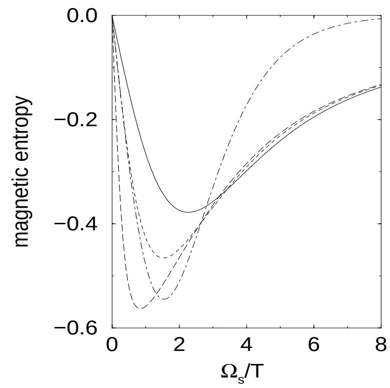

Fig. 3. Magnetic entropy $(\partial S(T,\Omega_s)/\partial\Omega_s)T/[c_V g\gamma_2]$ *vs.* Ω_s/T. The full, dashed and long-dashed lines are numerical results for $\gamma_2 = 1, 3, 10$. The dashed-dotted line is the magnetic entropy for free local moments, where the density of local moments has been rescaled in order to fix the slope at small magnetic field.

$-(\partial S/\partial H)_T$ will control the rate of adiabatic "heating" and "cooling" of the conduction electrons in a time varying magnetic field. The temperature hysteresis can then be made visible, *e.g.*, by measuring the magnetoresistance $\rho(H(t),T(t))$. The change of temperature of the electrons is $T - T_0 = -(1/k)T(\partial S/\partial H)_T(dH/dt) - (c_V(T,H)/k)(dT/dt)$, where T_0 is the temperature of the heat bath and k is a parameter describing the thermal coupling of the electrons with the bath. By using $\Delta c_V(T,H)$ and $\Delta S(T,H)$, one may estimate the order of magnitude of the hysteretic effect. Considering parameters which are in the domain of validity of the theory and correspond to the metallic phase, we take $g = 0.05$, $\gamma_2 = 3, 10$ and $Z \simeq 1$. With a sweep rate of 0.1 Tesla/minute and a thermal cooling of the order $k/c_V \approx 1/100$ s we obtain a temperature variation of few (5–15%) percent indicating that the effect we are describing should be visible under the above parameter conditions.

Concerning the size of the effects discussed in this paper and the possibility to observe them in a broader parameter range one should keep in mind the following.

i) Equations (1, 2) have been derived by perturbation theory to lowest order in g (metallic limit).

Indeed we can extend the region of validity of these expressions (and of Δc_V and ΔS) by assuming that g, γ_2 and Z are the running couplings of the RG analysis. In this case these couplings become temperature and field dependent and can assume large values, in particular $Z \gg 1$ even though $g\gamma_2 \leq 1$, which is the "optimistic" limit of confidence of the RG analysis [5,6]. This means that the predicted size of the hysteretic effect can be larger than the above conservative estimates.

ii) For $\Omega_s < T$ the running couplings should renormalize according to the RG equations derived at $H = 0$ and are therefore only functions of the temperature. This implies that a measurement of Δc_V and of ΔS at small H is a measurement of $\gamma_2(T)$ and $Z(T)$. In particular we get $\Delta c_V = -\Delta S = (1/2)gN_0TZ\gamma_2(1+\gamma_2)(\Omega_s/T)^2$ at leading order in $(H/T)^2$. It would be useful to measure these quantities in the metallic side of 2d systems showing a MIT to assess the validity of the interacting scaling theory of the 2d metallic behavior [6].

iii) The occurrence of a local moment instability at finite length makes relevant the sample dishomogeneity. Small local variations of disorder could result into regions with local moments coexisting with "more" metallic regions. Here, with regions of local moments we mean regions where the interplay of disorder and interaction in the spin channel has reached the strong coupling limit $g\gamma_2 \geq 1, Z \gg 1, \chi/\chi_0 \gg 1$. On a physical ground, one indeed expects that the weak coupling RG description of local moments in the metallic phase will evolve at low temperature into a more localized picture similar to that of the insulating phase [19], possibly in terms of an effective Kondo Hamiltonian [10]. However a comprehensive theory of this crossover from weak to strong coupling is still lacking and is a main open problem. A systematic analysis of the validity and of the breakdown of the existing scaling theory of disorder and interaction can shed light on this issue.

We like finally to comment on 3d systems. The 2d results in equations (1, 2) are easily extended to $d > 2$. Δc_V and ΔS show non monotonic behavior as function of T and Ω_s analogous to that in 2d. The leading $(H/T)^2$ contribution to Δc_V and ΔS now reads $\Delta c_V = 0.23gZ^{3/2}(1+\gamma_2)^{3/2}(\sqrt{1+\gamma_2}-1)N_0T\sqrt{T/(\hbar D)}(\Omega_s/T)^2$. and $\Delta S = -2\Delta c_V$. By assuming that this expression for Δc_V also holds approaching the strong coupling regime, where both χ/χ_0 and c_V/c_V^0 diverge, one obtains $(\Delta c_V/(\Omega_s/T)^2) \sim (\chi/\chi_0)^2(c_V^0/c_V)^{1/2}T^{3/2}$. Also this result can be tested experimentally to fix the limits of validity of the weak coupling description of local moments in 3d systems.

C.C. thanks M. Fabrizio for various helpful discussions. This work was supported by the TMR program of the European Union (P.S.).

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16. We recall that the interaction amplitude γ_2 is related to the Landau parameter F_a^0 *via* $\gamma_2 = -A_a^0 = -F_a^0/(1+F_a^0)$. We also note that in a magnetic field one should allow for two different running couplings in the triplet channels with $M = 0$ and $M = \pm 1$. This will become important when considering the crossover from metal to insulator induced by the magnetic field. Because the present discussion is mainly limited to small magnetic field and/or weak disorder, we will safely ignore this fact in the subsequent analysis.
17. Notice that equation (3.22) of reference [14] contains a minor error which we correct in equation (2).
18. Note that by making use of equation (5) and the small field expansion for $f_2(x)$ one obtains the magnetic field dependent correction to the thermodymamic potential in the form $-(1/2)gN_{QP}\gamma_2(1+\gamma_2)\Omega_s^2 \log T\tau$ which reproduces the perturbative renormalization of the spin susceptibility given in reference [7].
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